ON THE PROBABLE REASON WHY CERTAIN
PERIODIC COMETS HAVE NOT BEEN FOUND
ON THEIR PREDICTED RETURNS

by

Jessica M. Young

Submitted in partial fulfilment of the requirements
the degree of Doctor of Philosophy.

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Certain comets for which periodic orbits were derived have not been found again in spite of the fact that search ephemerides have been computed for expected returns. In some cases a satisfactory explanation has been made, while in others the failure to find the comet has remained unexplained. The orbits from which the ephemerides are computed are in general definitive orbits based upon a least squares reduction of normal places formed from all the available observations. In the computation of a search ephemeris allowance should be made for uncertainties in the elements due to errors in the observations, limitations in the length of the arc over which the comet was observed, and partial theoretical indeterminateness. In some cases allowance has been made and still the comets were not found. The question arises whether perhaps because of the inaccuracies in the observations and the inconsistencies between them, it would have been advisable not to go to the laborious work of forming normal places and making a least squares reduction, but rather to compute a set of elements based upon three reliable positions covering as long an arc as possible. Each of these three positions could be formed from three or four observations close together in time and shown by comparison with an ephemeris to agree fairly well with each other. In order to furnish criteria for determining what variations can be allowed in the elements because of the inaccuracy of the observational material, the period can be varied arbitrarily and corresponding new orbits

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computed. A comparison of the changes in α and δ for given observation dates due to this arbitrary change of the Period with the uncertainty of the given observations would furnish the criteria desired. This method of procedure will give a means of determining whether the search ephemeris gives sufficient range in which to search for the comet. If the range is larger than that allowed for in the ephemeris, the failure to find the comet would be explained by the fact that it probably lay in a region outside of the limits of the search ephemeris.

For example, a search ephemeris was computed for Brorsen's comet (1879 I) for its return in 1900. Only a small range of $\pm$ 4 days was allowed in the perihelion time. This small range produced a range in α of as much as 31° and in δ of as much as 8°. A search was made by E.C.Pickering[1] but the comet failed to appear.

1 A.N. 155, p. 247

The range of the comet's position is probably much larger, a variation of only four days being rather small.

On the other hand Comet d 1913 (Delavan) which was later found to be identical with Comet 1852 ~~1852~~ IV (Westphal), was discovered in a region covered by Hnatek's ephemerides[2] for the return

2 A.N. 193, p. 205.

of Westphal's comet in 1913. These ephemerides are computed for periods 60.5, 60.6, etc. years up to 61.3 years. Thus according to these ephemerides on January 25 the comet could lie anywhere in a region 29° in right ascension by 37° in declination, and on September 22 anywhere in a region 87° by 118°. The positions of Delavan's

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comet were found to correspond to positions of Westphal's comet for a period 61.121 years. The computation of elements based upon this period and the 1913 observations resulted in elements closely agreeing with those of Westphal's comet and thus proved the identity.[1]

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[1] L.O.Bulletin No. 244

In this paper the method of variation of period will be applied to Comets 1884 II (Barnard) and 1881 V (Denning). The work will be based upon three of the places used by the computer of what appears to be the best orbit. The period of this orbit will be arbitrarily varied and differential corrections found to the heliocentric velocities and coordinates and the selected observations represented to obtain the residuals. These residuals will then be studied to obtain the range of solution.

The differential corrections will be obtained by means of Leuschner's formulae for a Differential Correction.[2] In these

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[2] Publ. L.O. Vol VII.

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[1] p. 295.

$$P = \frac{1}{2a_\circ} - (x_\circ' P_x + y_\circ' P_y + z_\circ' P_z)$$

$\frac{1}{a_\circ} - \frac{1}{a}$ must be substituted for $\frac{1}{a_\circ}$. This method was devised in order to determine from very short arcs fairly accurate elliptic orbits in cases of comets whose identity with previously observed comets seemed probable. In fact, it has been of considerable service in establishing identities. It was applied, for instance, in the case of Comet b 1912 (Schaumasse-Tuttle).

"A preliminary orbit of Comet b 1912, discovered by Schaumasse, was calculated by Fayet at Nice before the observations necessary for a computation had reached Berkeley. Fayet announced a similarity of the parabolic elements with those of Tuttle's comet, which had appeared 13 1/2 years previously. A new process, in the nature of a conditioned solution, was then applied by us. The interval between the perihelion passages of the two comets which are suspected to be identical is assumed to be a multiple of the period. In order to test this new principle, a general solution without hypothesis, ... was . . . applied. A remarkable

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condition was found to exist. The first three observations made on October 21, 22 and 23 could be satisfied by any kind of an orbit from a circle to a parabola. If the new principle had the practical value that the theory showed it to possess, then a conditioned solution of the orbit of Schaumasse's comet, on the basis of the actual period of 13.7 years of Tuttle's comet, should bring the new elements into close agreement with those of Tuttle's. This was actually found to be the case, while neither a parabolic nor a general solution could have confirmed the identity of the two comets from so short an arc. The introduction of this principle was all the more important for this comet because an attempt to reproduce the position of the new comet from the orbit of Tuttle's comet resulted in a discrepancy of 80° in the position. This discrepancy was due partly to perturbations which Tuttle's comet had suffered in the meantime and partly to the relative positions of Sun, comet and Earth, which aggravated any displacement with reference to the Sun when viewed from the Earth. The interval between the dates of perihelion passage in 1899 and 1912 corresponded to an average mean motion of 263". Later Fayet calculated the effect of perturbations on the original mean motion of 269".6 and found this effect to change it to 264", in close agreement with the value we had obtained by our principle of identification without performing the computation of the perturbations."[1]

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1 "Recent Progress in the Study of Motions of Bodies in the Solar System" - A.O.Leuschner in The Adolfo Stahl Lectures in Astronomy.

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Metcalf's comet from an arc of 14 days which agreed very closely with those of Brorsen's comet, 1847 V, and which also agreed very closely with those computed later from a larger arc of 37 days by Braae and Fischer-Petersen.[2]

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The same method proved equally adaptible in the case of Comet d 1919 (Finlay-Sasaki).[3]

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These solutions are usually based on very short arcs and the series expressions for "f" and "g" are sufficiently convergent both for the direct solution and for the differential correction, the only modification being the one mentioned above. In the cases under investigation here, on the other hand, the conditioned solution has to be based upon long arcs which require the use of closed expressions. Professor Leuschner gives the fundamental equations for this on page 265, namely

$$a_i \partial\rho_0 + b_i \partial x_0' + c_i \partial y_0' + d_i \partial z_0' = n_i$$

or as on page 334

$$\alpha_i \partial\rho_0 + \beta_i \partial x_0' + \gamma_i \partial y_0' + \delta_i \partial z_0' = v_i \qquad \text{where } i = 1,2,3,4,$$

the coefficients being expressed by the quantities given on page 334 and applying to the case of an initial elliptic orbit. These

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are four equations corresponding to the residuals in the α and δ of the first and third places and when solved give corrections to ρ_0, x_0', y_0', and z_0' which would yield a general orbit accurately representing the observations. If, however, one desires to obtain a conditioned orbit in which one element is known (in this case the Period or the semi-major axis a), he must use just three of the above equations and substitute the assumed condition for the other equation. To force this condition it is necessary to compute P_x, P_y, P_z, Q_x, Q_y, and Q_z from the above equations and then apply formulae (5) and (6) p.295 to determine $\partial\rho_0$ with the substitution of $\frac{1}{a_0} - \frac{1}{a}$ for $\frac{1}{a_0}$. For the case where the initial orbit is parabolic the fundamental equations are given in (47) p. 302 and the P's and Q's in (54) and (55) p. 303. The coefficients in (47) p. 302 differ from those in the general formulae p.334 by the terms with subscript "$\overset{m}{M}$". Therefore the formulae for a conditioned solution based upon an initial ellipse might be derived as on p. 303. For the sake of clearness, however, they have been derived directly from p. 334 as follows:

$$\beta_1 \partial x_0' + \gamma_1 \partial y_0' + \delta_1 \partial z_0' = v_1 - \alpha_1 \partial\rho_0$$

$$\beta_3 \partial x_0' + \gamma_3 \partial y_0' + \delta_3 \partial z_0' = v_3 - \alpha_3 \partial\rho_0$$

$$\beta_4 \partial x_0' + \gamma_4 \partial y_0' + \delta_4 \partial z_0' = v_4 - \alpha_4 \partial\rho_0$$

Here the equation subscript 2 has been discarded so as to throw the residual into the first declination.

Solving these by determinants gives

$$\partial x_0' = \frac{1}{\Delta}\begin{vmatrix} v_1-\alpha_1\partial\rho_0 & \gamma_1 & \delta_1 \\ v_3-\alpha_3\partial\rho_0 & \gamma_3 & \delta_3 \\ v_4-\alpha_4\partial\rho_0 & \gamma_4 & \delta_4 \end{vmatrix}, \qquad \Delta = \begin{vmatrix} \beta_1 & \gamma_1 & \delta_1 \\ \beta_3 & \gamma_3 & \delta_3 \\ \beta_4 & \gamma_4 & \delta_4 \end{vmatrix}$$

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with corresponding expressions for $\partial y_0'$ and $\partial z_0'$.

Expanding these gives

$$\Delta = \Sigma \beta_i b_i$$

$$\partial x_0' = \frac{\Sigma \nu_i b_i}{\Delta} - \frac{\Sigma \alpha_i b_i}{\Delta}\partial \rho_0 = P_x - Q_x \partial \rho_0$$

$$\partial y_0' = \frac{\Sigma \nu_i c_i}{\Delta} - \frac{\Sigma \alpha_i c_i}{\Delta}\partial \rho_0 = P_y - Q_y \partial \rho_0$$

$$\partial z_0' = \frac{\Sigma \nu_i d_i}{\Delta} - \frac{\Sigma \alpha_i d_i}{\Delta}\partial \rho_0 = P_z - Q_z \partial \rho_0$$

where i = 1,3,4 and

$$b_1 = \gamma_3\delta_4 - \gamma_4\delta_3 \qquad c_1 = \beta_4\delta_3 - \beta_3\delta_4 \qquad d_1 = \beta_3\gamma_4 - \beta_4\gamma_3$$

$$b_3 = \gamma_4\delta_1 - \gamma_1\delta_4 \qquad c_3 = \beta_1\delta_4 - \beta_4\delta_1 \qquad d_3 = \beta_4\gamma_1 - \beta_1\gamma_4$$

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Then by equating coefficients one gets

$$P_x = \frac{\Sigma \nu_i b_i}{\Delta} \qquad Q_x = \frac{\Sigma \alpha_i b_i}{\Delta}$$

$$P_y = \frac{\Sigma \nu_i c_i}{\Delta} \qquad Q_y = \frac{\Sigma \alpha_i c_i}{\Delta}$$

$$P_z = \frac{\Sigma \nu_i d_i}{\Delta} \qquad Q_z = \frac{\Sigma \alpha_i d_i}{\Delta}$$

The P's and Q's may now be used in (5) and (6) p. 295 together with the substitution of $\frac{1}{a_0} - \frac{1}{a}$ for $\frac{1}{a_0}$. If it is desired to concentrate the residual in the third declination the formulae remain the same as above except that 2 is substituted for subscript 3, and 3 for 4.

Comet 1884 II. (Barnard)

Comet 1884 II (Barnard) was discovered by Barnard in Nashville, Tennessee, on July 16, 1884. The comet seemed like a rather large, faint nebula with a slightly condensed nucleus. It remained under observation until November 20, there being in all 288 observations. The first five orbits computed by Chandler, Weiss, Oppenheim, Stechert, and Ravené, respectively were parabolic and did

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$$P_x = \frac{\Sigma v_i b_i}{\Delta} \qquad Q_x = \frac{\Sigma \alpha_i b_i}{\Delta}$$

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[1] Verzeichniss der Elemente der bisher berechneten Cometenbahnen nebst Anmerkungen und Literatur-Nachweisen."

T B.M.T. 1884	ω	☊	i	e	P	Arc	Comp
.16.54671	301°09'39"	4°54'01"	5°30'36"	0.595879	5.6618	July 17-Sept. 17	Fin
16.46276	300 57 44	5 11 24	5 27 19	0.5823692	5.3618	July 25-Sept 23	Mor
16.32674	300 46 18	5 23 51	5 24 49	0.571626	5.1435	Aug. 12-Oct. 20	Fri
16.52005	301 02 42	5 08 38	5 27 33	0.5839537	5.3945	July 23-Oct. 24	Egb
16.52067	301 03 41	5 03 50	5 28 50	0.588663	5.5	July 23-Sept. 14	Ber
16.48480	300 59 46	5 10 52	5 27 14	0.582461	5.3632	July 26-Oct. 23	
16.51654	301 01 59	5 08 59	5 27 38	0.5842139	5.4037	July 23-Nov. 12	

Mean equinox 1884.0.

The last is a definitive orbit and is based upon all the observations available.[2]

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[2] A.N. 123 p. 145.

No search ephemeris was computed for 1890 because the comet would be too close to the Sun and therefore in an unfavorable position for observation. For the year 1895, however, a search ephemeris was computed by Berberich.[3] This ephemeris extends from April 24 to

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[3] A.N. 136 p. 333 and 138 p. 287.

October 1, 1895 and the elements used for it are the last given above corrected for the Jupiter perturbations which are very small. Berberich

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T M.T 4	ω	Ω	i	e	P	Arc	Comp
4671	301°09'39"	4°54'01"	5°30'36"	0.593879	5.3618	July 17-Sept. 17	Fi
6276	300 57 44	5 11 24	5 27 19	0.5823692	5.3618	July 25-Sept 23	Mor
2674	300 46 19	5 23 51	5 24 49	0.571626	5.1435	Aug. 12-Oct. 20	Fr
2005	301 02 42	5 08 38	5 27 33	0.5859537	5.3943	July 23-Oct. 24	Eg
2067	301 03 41	5 03 50	5 29 50	0.588663	5.5	July 23-Sept. 14	Ber
8480	300 59 46	5 10 52	5 27 14	0.582461	5.3632	July 26-Oct. 25	
1654	301 01 59	5 08 59	5 27 38	0.5862139	5.4037	July 23-Nov. 12	

Mean equinox 1884.0.

The last is a definitive orbit and is based upon all the observations available.[2]

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[2] A.N. 123 p. 145.

No search ephemeris was computed for 1890 because the comet would be too close to the Sun and therefore in an unfavorable position for observation. For the year 1895, however, a search ephemeris was computed by Berberich.[3] This ephemeris extends from April 24 to

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[3] A.N. 136 p. 333 and 138 p. 287.

October 1, 1895 and the elements used for it are the last given above corrected for the Jupiter perturbations which are very small. Berberich

assumed that the time of Perihelion passage at this second return might be in error by ± 8 days which corresponds to ± 4 days in the period for each of the two returns in 1890 and 1895 and allowed for this by computing three ephemerides, one for T as given by the orbit, one for T increased by 8 days, and one for T decreased by 8 days, the other elements remaining unchanged. There is no record of the comet having been found or searched for on this return. Berberich also computed an ephemeris for a return in 1900, the ephemeris extending from August 28 to December 2.[1] In this ephemeris he allows

[1] A.N. 153 p. 221

the perihelion date an increased range of 32 days, or ±16 days instead of 24 days or ± 12 days. On August 28 the comet according to the search ephemeris could lie anywhere within an area 11°15' by 5°5' and on December 2 within an area 18°30' by 4°59'. E.C.Pickering searched for the comet on this return, but states that he was unable to find it.[2]

[2] A.N. 155 p. 247.

The work of Berberich is a very precise, detailed piece of work. He first makes an independent investigation of the comparison stars including their proper motion. Then with the second set of elements computed by himself he computes an ephemeris covering the whole interval of observation. The observations are listed chronologically and the residuals show the difference between the observed places and the places as computed from the ephemeris. These observations are weighted and grouped into twelve groups and the weighted mean residual formed for each group. With these weighted mean resi-

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duals and positions from the ephemeris 12 normal places are formed. The perturbations in α and δ due to the Earth and Jupiter are then tabulated and applied to the normal places. Then a least squares reduction is made with the use of Schönfeld's Differential formulae. The equations of condition are combined into normal equations in which ξ, a quantity depending upon $d\mu$ is the unknown. These are then solved by least squares to determine ξ and the corrections to the elements found. The resulting elements after the application of these corrections are as follows:

Epoch B.M.T. 1884 August 16.5

$$M_0 = 359^\circ\ 59'\ 49.''13 + 2.50\ d\mu$$

$$\left.\begin{aligned} \omega &= 301\ 01\ 58.63 - 21.10\ d\mu \\ \Omega &= 5\ 08\ 59.12 + 26.44\ d\mu \\ i &= 5\ 27\ 38\ 40 - 5.53\ d\mu \end{aligned}\right\} 1884.0$$

$$\varphi = 35\ 44\ 50.92 - 98.25\ d\mu$$

$$\mu = 657''.0839 \pm 0''.8876$$

$$\log a = 0.4882572$$

$$T = 1884 \text{ August } 16.516543$$

$$P = 1972.35 \pm 2.66 \text{ days.}$$

This resulting orbit agrees quite closely with the orbit by Egbert, the mean motion of the latter orbit being 657.''75. It seems that Egbert's orbit would have been satisfactory for the computation of a search ephemeris and that the formation of normal places and a least squares reduction was really unnecessary. At least some elements computed from three places each formed from three or four observations which agree fairly closely with each other should have

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furnished an idea as to the limits of position.

On looking over the tabulation of the disagreement between ephemeris and observation on p. 169 and following, it is seen that there is a great discrepancy between observations. The residuals oscillate back and forth and do not show much of a systematic tendency at all. For example, on September 14 an observation at Dresden gives $d\alpha = +0^s.19$ and $d\delta = +7.''5$, while one at Arcetri gives $-0^s.23$ and $-6.''8$. The general run of residuals in that vicinity is toward that of the Arcetri residuals. The variation in residuals in δ is seen in this case to be $14.''3$. Likewise on September 21 Berlin gives $d\alpha = -0^s.82$, $d\delta = -4.''5$ and the Cape of Good Hope gives $+0^s.50$ and $-4.''0$. Here the run of residuals is also negative, the $+0^s.50$ being a decided departure. Here there is a large range in $d\alpha$ amounting to $1^s.32$. On September 11 Nice and Glasgow show a difference of $1^s.34$ in $d\alpha$, the Glasgow observation departing decidedly from the general run. On October 15 there is a variation in $d\alpha$ of $1^s.98$, in $d\delta$ $13.''8$. On October 9 the range of $d\delta$ is $14.''5$, and on October 16 it is $21.''4$.

Berberich recognized the inaccuracy of the observations and allowed for it in the search ephemeris by allowing his perihelion time a range of 16 days. This range will now be investigated to see whether this allowance was large enough in the face of the inaccuracy of the available material.

The following three of Berberich's places will be used.

	Date (B.M.T.)	α (app.)	δ (app.)
I.	July 23.5	$240^\circ\ 53'\ 27''.4$	$-37^\circ\ 14'\ 07''.2$
II.	Aug. 16.5	259 20 38 .2	36 20 53 .0
III.	Nov. 12.5	335 03 06. 3	$-$ 9 36 32 .4

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	Date (G.M.T.)	α (app.)	δ (app.)
I.	July 23.5	240° 53' 27".4	−37° 14' 07".2
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The middle place is the epoch of Berberich's orbit and the other two are so chosen as to cover the whole arc of observation. It is recognized that a better choise of places could have been made - namely, one in which the intervals of time between observations are equal. However, the work is to be approximate and need not necessarily be based upon equal intervals. These times have been corrected for aberration and the places for parallax. Berberich's orbit leaves the following residuals in the first and third normal places:

		I	III
(O - C)	$\partial\alpha\cos\delta$	+13".1	-1".9
(O - C)	$\partial\delta$	+ 2".6	-13".2

These are residuals from a least squares reduction and therefore should give the best representation possible on the assumption that the sum of the squares of the residuals should be a minimum. Therefore the residuals will be considered errors of observation and assumed equal to zero and a differential correction made of Berberich's orbit by Leuschner's method of Conditioned Solution assuming periods 5.3 and 5.5 years. The resulting residuals, after correction has been made for the perturbations as tabulated by Berberich, are

Period		5.3		Berberich		5.5	
		I	III	I	III	I	III
(O - C)	$\partial\alpha\cos\delta$	+14".4	-2".0	+13".1	-1".9	+12".7	- 1".1
(O - C)	$\partial\delta$	+23 .5	-12 .3	+ 2 .6	-13 .2	-17 .0	-13.6

The residuals in the first and third right ascensions and the third declination are practically the same as those of Berberich's orbit and furnish a check on the computation. The slight differences are accounted for by the effect of higher order terms in the differential correction and by the accumulation of errors in the computation

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Period	5.3		Berberich		5.5	
	I	III	I	III	I	III
(O - C) $\alpha \cos \delta$	+ 14".4	-2".0	+13".1	-1".9	+ 12".7	- 1".1
(O - C) δ	+ 25 .5	-12 .3	+ 2 .6	-13 .2	-17 .0	-13.6

The residuals in the first and third right ascensions and the third declination are practically the same as those of Berberich's orbit and furnish a check on the computation. The slight differences are accounted for by the effect of higher order terms in the differential correction and by the accumulation of errors in the computation

to six places as compared with Berberich's seven-place computation. The differential corrections were made so as to confine the residual to the first declination and this residual alone is of importance for our purpose. By interpolation between the residuals in the first declination for the periods 5.3 and 5.5 years, it is found that a residual of 2"6 corresponds to a period of 5.4032 years which is seen to agree very closely with the period of Berberich's orbit.

To ascertain how large a range to allow in $\partial\delta_1$ it is necessary to find some relation between $\partial\delta_1$ and $\partial\delta_{III}$, $\partial_1\alpha_1$, and $\partial_1\alpha_{III}$. The range in $\partial\delta_1$ will be due not only to the range in δ_1 itself but also to the range in α_1, α_{III}, and δ_{III} because the outstanding residual is thrown into the first declination.

In any orbit the following relations hold:[1]

[1] Publ. L.O. Vol. VII p. 322.

$P_{z_1} - Q_{z_1}\partial\rho_0 = P_{z_{III}} - Q_{z_{III}}\partial\rho_0 = \partial z_0'$, where $\partial z_0'$ is the correction to z_0' necessary to reduce $\partial\delta_1$, $\partial\delta_{III}$, $\partial_1\alpha_1$, and $\partial_1\alpha_{III}$ to zero.

$$P_{z_1} = \frac{\partial\delta_1 + C_1 \sin\delta_1 (\cos\alpha_1 P_x + \sin\alpha_1 P_y)}{C_1 \cos\delta_1}$$

$$P_{z_{III}} = \frac{\partial\delta_{III} + C_{III} \sin\delta_{III} (\cos\alpha_{III} P_x + \sin\alpha_{III} P_y)}{C_{III} \cos\delta_{III}}$$

$$Q_{z_1} = \frac{B_1 + C_1 \sin\delta_1 (\cos\alpha_1 Q_x + \sin\alpha_1 Q_y)}{C_1 \cos\delta_1}$$

$$Q_{z_{III}} = \frac{B_{III} + C_{III} \sin\delta_{III} (\cos\alpha_{III} Q_x + \sin\alpha_{III} Q_y)}{C_{III} \cos\delta_{III}}$$

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To ascertain how large a range to allow in $\delta\delta_{1}$ it is necessary to find some relation between $\delta\delta_{1}$ and $\delta\delta_{111}$, $\delta\alpha_{1}$, and $\delta\alpha_{111}$. The range in $\delta\delta_{1}$ will be due not only to the range in δ_{1} itself but also to the range in α_{1}, α_{111}, and δ_{111} because the outstanding residual is thrown into the first declination.

In any orbit the following relations hold:[1]

[1] Publ. L.O. Vol. VII p. 322.

$P_{2_1} - Q_{2_1}\delta\rho_0 = P_{2_{111}} - Q_{2_{111}}\delta\rho_0 = \delta z_0'$, where $\delta z_0'$ is the correction to z_0' necessary to reduce $\delta\delta_{1}$, $\delta\delta_{111}$, $\delta\alpha_{1}$, and $\delta\alpha_{111}$ to zero.

$$P_{2_1} = \frac{\delta\delta_1 + C_1 \sin\delta_1 (\cos\alpha_1 P_x + \sin\alpha_1 P_y)}{C_1 \cos\delta_1}$$

$$P_{2_{111}} = \frac{\delta\delta_{111} + C_{111} \sin\delta_{111} (\cos\alpha_{111} P_x + \sin\alpha_{111} P_y)}{C_{111} \cos\delta_{111}}$$

$$Q_{2_1} = \frac{B_1 + C_1 \sin\delta_1 (\cos\alpha_1 Q_x + \sin\alpha_1 Q_y)}{C_1 \cos\delta_1}$$

$$Q_{2_{111}} = \frac{B_{111} + C_{111} \sin\delta_{111} (\cos\alpha_{111} Q_x + \sin\alpha_{111} Q_y)}{C_{111} \cos\delta_{111}}$$

$$P_x = \frac{C_{111}\cos\alpha_{111}\,\partial_1\alpha_1 - C_1\cos\alpha_1\,\partial_1\alpha_{111}}{C_1 C_{111}\sin(\alpha_{111}-\alpha_1)} \qquad Q_x = \frac{A_1 C_{111}\cos\alpha_{111} - A_{111} C_1\cos\alpha_1}{C_1 C_{111}\sin(\alpha_{111}-\alpha_1)}$$

$$P_y = \frac{C_{111}\sin\alpha_{111}\,\partial_1\alpha_1 - C_1\sin\alpha_1\,\partial_1\alpha_{111}}{C_1 C_{111}\sin(\alpha_{111}-\alpha_1)} \qquad Q_y = \frac{A_1 C_{111}\sin\alpha_{111} - A_{111} C_1\sin\alpha_1}{C_1 C_{111}\sin(\alpha_{111}-\alpha_1)}$$

Substituting these quantities in the first equation above and solving for $\partial\delta_1$ gives

$$\partial\delta_1 = I\,\partial\delta_{111} + M_3\,\partial_1\alpha_1 + M_1\,\partial_1\alpha_{111} + K\,\partial\rho_0$$

where

$$I = \frac{C_1\cos\delta_1}{C_{111}\cos\delta_{111}}$$

$$M_1 = \frac{C_1}{C_{111}}\left[m_1 - m_3\cos(\alpha_{111}-\alpha_1)\right] \qquad m_1 = \frac{\sin\delta_1}{\sin(\alpha_{111}-\alpha_1)}$$

$$M_3 = m_3 - m_1\cos(\alpha_{111}-\alpha_1) \qquad m_3 = \frac{\cos\delta_1}{\sin(\alpha_{111}-\alpha_1)}\cdot\tan\delta_{111}$$

$$\left[A_1 M_3 + A_{111} M_1 + B_{111} I - B_1\right] = -\left\{\frac{f_1}{\rho_1}\left[M_3\sin(\alpha_{111}-\alpha_1)\cos\delta_{11} - \sin(\delta_{11}-\delta_1)\right] + \frac{f_{111}}{\rho_{111}}\left[M_1\sin(\alpha_{11}-\alpha_{111})\cos\delta_{11} + I\sin(\ldots\right.\right.$$

provided that the middle place is assumed to be accuratedy given.

A_1, A_{111}, C_1, C_{111}, B_1, B_{111} have the values given on p. 321 and f_1, f_{111} are the expressions from the original orbit. In the last expression for K, the approximate expressions an p.266 which are applicable only for short arcs have been substituted, namely:

$$A_1 = \frac{f_1}{\rho_1}\sin(\alpha_{11}-\alpha_1)\cos\delta_{11} \qquad B_1 = \frac{f_1}{\rho_1}\sin(\delta_{11}-\delta_1)$$

$$A_{111} = \frac{f_{111}}{\rho_{111}}\sin(\alpha_{11}-\alpha_{111})\cos\delta_{11} \qquad B_{111} = \frac{f_{111}}{\rho_{111}}\sin(\delta_{11}-\delta_{111})$$

Before determining the range in $\partial\delta_1$ it was deemed advisable to differentially correct Berberich's orbit by Leuschner's method on the basis of the three normal places used here. This differential correction yielded the following residuals:

$$P_x = \frac{C_{III}\cos\alpha_{III}\,\delta\alpha_1 - C_1\cos\alpha_1\,\delta\alpha_{III}}{C_1 C_{III}\sin(\alpha_{III}-\alpha_1)} \qquad Q_x = \frac{A_1 C_{III}\cos\alpha_{III} - A_{III} C_1\cos\alpha_1}{C_1 C_{III}\sin(\alpha_{III}-\alpha_1)}$$

$$P_y = \frac{C_{III}\sin\alpha_{III}\,\delta\alpha_1 - C_1\sin\alpha_1\,\delta\alpha_{III}}{C_1 C_{III}\sin(\alpha_{III}-\alpha_1)} \qquad Q_y = \frac{A_1 C_{III}\sin\alpha_{III} - A_{III} C_1\sin\alpha_1}{C_1 C_{III}\sin(\alpha_{III}-\alpha_1)}$$

Substituting these quantities in the first equation above and solving for $\delta\delta_1$ gives

$$\delta\delta_1 = I\,\delta\delta_{III} + M_3\,\delta\alpha_1 + M_1\,\delta\alpha_{III} + K\,\delta\rho_0$$

where

$$I = \frac{C_1\cos\delta_1}{C_{III}\cos\delta_{III}}$$

$$M_1 = \frac{C_1}{C_{III}}\left[m_1 - m_3\cos(\alpha_{III}-\alpha_1)\right] \qquad m_1 = \frac{\sin\delta_1}{\sin(\alpha_{III}-\alpha_1)}$$

$$M_3 = m_3 - m_1\cos(\alpha_{III}-\alpha_1) \qquad m_3 = \frac{\cos\delta_1}{\sin(\alpha_{III}-\alpha_1)}\cdot\tan\delta_{III}$$

$$K = [M_3 A_1 + A_{III} M_1 + B_{III} I - B_1] = -\left\{\frac{f_1}{\rho_1}\left[M_3\sin(\alpha_{II}-\alpha_1)\cos\delta_{II} - \sin(\delta_{II}-\delta_1)\right] + \frac{f_{III}}{\rho_{III}}\left[M_1\sin(\alpha_{II}-\alpha_{III})\cos\delta_{II} + I\ldots\right.\right.$$

provided that the middle place is assumed to be accurately given.

A_1, A_{III}, C_1, C_{III}, B_1, B_{III} have the values given on p. 321 and f_1, f_{III} are the expressions from the original orbit. In the last expression for K, the approximate expressions on p.256 which are applicable only for short arcs have been substituted, namely:

$$A_1 = \frac{f_1}{\rho_1}\sin(\alpha_{II}-\alpha_1)\cos\delta_{II} \qquad B_1 = \frac{f_1}{\rho_1}\sin(\delta_{II}-\delta_1)$$

$$A_{III} = \frac{f_{III}}{\rho_{III}}\sin(\alpha_{II}-\alpha_{III})\cos\delta_{II} \qquad B_{III} = \frac{f_{III}}{\rho_{III}}\sin(\delta_{II}-\delta_{III})$$

Before determining the range in $\delta\delta_1$ it was deemed advisable to differentially correct Berberich's orbit by Leuschner's method on the basis of the three normal places used here. This differential correction yielded the following residuals:

	I	III
(O - C) $\partial\alpha \cos\delta$	$-2''.1$	$-1''.2$
(O - C) $\partial\delta$	$+0''.2$	$-3''.1$

It seems, however, that this orbit would be preferable to the least squares orbit for the prediction of future places of the comet because it holds approximately for the extreme limits of the arc of observation while the least squares orbit has its largest residuals at the ends of the arc. The period of this orbit is 5.3905 years corresponding to a mean motion of 658".23.

The equation for $\partial\delta_1$ for this comet is

$$\partial\delta_1 = -0.620\,\partial\delta_{III} - 0.117\,\partial\alpha_{I} + 0.460\,\partial\alpha_{III} - 1''.5$$

The last term is the value of $K\,\partial\rho_0$ corresponding to the $\partial\rho_0$ required to remove the residuals of the orbit of period 5.3905 years.

The limiting values for the residuals of the observations combined by Berberich in the formation of the normal places are

$$\partial\alpha_I\begin{cases}+1^s.49\\-6^s.70\end{cases}\quad \partial\alpha_{III}\begin{cases}-0^s.49\\-0^s.87\end{cases}\quad \partial\delta_I\begin{cases}+11''.5\\-3''.9\end{cases}\quad \partial\delta_{III}\begin{cases}-6''.9\\-18''.8\end{cases}$$

The values used by Berberich to form the normal places are

$\partial\alpha_I = +1^s.35$, $\partial\alpha_{III} = -0^s.62$, $\partial\delta_I = +7''.6$, $\partial\delta_{III} = -11''.9$.

Therefore the departure of the limiting observations from the normal places used is as follows:

$$\partial\alpha_I\begin{cases}+0^s.14 = +2''.1\\-2^s.05 = -30''.8\end{cases}\quad \partial\alpha_{III}\begin{cases}+0^s.13 = +2''.0\\-0^s.25 = -3''.8\end{cases}\quad \partial\delta_I\begin{cases}+3''.9\\-11''.5\end{cases}\quad \partial\delta_{III}\begin{cases}+5''.0\\-6''.9\end{cases}$$

By representing these limiting values rather than the values used in the normal places the largest positive value and the largest negative value for $\partial\delta_1$ corresponding to the range of the individual observation will be found.

	I	III
$(O - C)\ \delta\alpha \cos\delta$	$-\ 2''.1$	$-1''.2$
$(O - C)\ \delta\delta$	$+\ 0''.2$	$-3''.1$

It seems, however, that this orbit would be preferable to the least squares orbit for the prediction of future places of the comet because it holds approximately for the extreme limits of the arc of observation while the least squares orbit has its largest residuals at the ends of the arc. The period of this orbit is 5.3905 years corresponding to a mean motion of 658".23.

The equation for $\delta\delta_I$ for this comet is

$$\delta\delta_I = -0.620\,\delta\delta_{III} - 0.117\,\delta\alpha_I + 0.460\,\delta\alpha_{III} - 1''.5$$

The last term is the value of $K\delta\rho_0$ corresponding to the $\delta\rho_0$ required to remove the residuals of the orbit of period 5.3905 years.

The limiting values for the residuals of the observations combined by Berberich in the formation of the normal places are

$$\delta\alpha_I\begin{cases}+1^s.49\\-0^s.70\end{cases}\qquad \delta\alpha_{III}\begin{cases}-0^s.49\\-0^s.87\end{cases}\qquad \delta\delta_I\begin{cases}+11''.5\\-\ 3''.9\end{cases}\qquad \delta\delta_{III}\begin{cases}-6''.9\\-18''.8\end{cases}$$

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By representing these limiting values rather than the values used in the normal places the largest positive value and the largest negative value for $\delta\delta_I$ corresponding to the range of the individual

For the largest positive value

$\partial\delta_1 = -0.620(-6''.9) - 0.117(-30''.8) + 0.460(2''.0) - 1''.5 + 3''.9$

or

$\partial\delta_1 = +11''.2$

For the largest negative value

$\partial\delta_1 = -0.620\ (5.''0) - 0.117(2.''1) + 0.460(-3.''8) - 1.''5 - 11.''5$

or $\partial\delta_1 = -18.''0.$

If the residuals in the first declination for periods 5.3, 5.4037, and 5.5 years are reduced to equal intervals of 0.1 year, it is found that the first differences are practically constant, an increase of 0.1 year amounting to a change of -20".3 in the residual. Therefore to remove the residuals in the other three places and force a residual of + 11".2 into the first declination the period of the above orbit would have to be changed by -0.0552 years, making the forced period 5.3353 years. To force the residual of -18".0 into the first declination in the same way, the period would have to be changed by + 0.0887 years making the period 5.4792 years. Therefore the extreme possible range of the period is 0.1439 years or 5252 days as compared with the range of 8 days allowed by Berberich in his ephemeris This would give an extreme range of 105 days instead of 16 days in 1895 and of 157.5 instead of 32 in 1900. The extreme range of 52.5 days is seen to include the orbits by Morrison, Egbert and two by Berberich.

The elements corresponding to the period 5.3905 years are

Epoch	1884	August	16.5	
M	359°	59'	46".3	
ω	301	01	06 .4	1884.0
☊	5	10	15. 3	1884.0
i	5	27	31 .1	1884.0
φ	35	42	55 .9	

for the largest positive value 17.

$\delta\delta_1 = -0.620(-6''.9) - 0.117(-30''.8) + 0.460(2''.0) - 1''.5 + 3''.9$

or

$\delta\delta_1 = +11''.2$

For the largest negative value

$\delta\delta_1 = -0.620\ (5''.0) - 0.117(2''.1) + 0.460(-3''.8) - 1''.5 - 11''.5$

or $\delta\delta_1 = -18''.0$.

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Epoch	1884	August 16.5		
M	359°	59'	46".3	
ω	301	01	06 .4	1884.0
Ω	5	10	15. 3	1884.0
i	5	27	31 .1	1884.0
φ	35	42	35 .9	

log a 0.487754

e 0.583761

μ 658".23

P 5.3905 years

If these elements are allowed to undergo the same changes due to a change $d\mu$ in μ as the changes tabulated by Berberich for his elements, the elements corresponding to the limiting periods are

Epoch	1884 August 16.5			1884 August 16.5		
P =	5.3353 years			5.4792 years		
M =	0°	00'	03".3	359°	59'	09".7
ω =	300	58	42 .7	301	04	51 .1
☊ =	5	13	15 .4	5	05	33 .7
i =	5	26	53 .4	5	28	30 .0
φ =	35	31	46 .8	36	00	22 .3
log a =	0.484773			0.492478		
μ =	665".04			647".58		

Equinox 1884.0

As a means of comparison with Berberich's ephemeris, positions of the comet were computed from each of these two extreme sets of elements for August 28 and December 2, 1900, the two outer dates of Berberich's search ephemeris. These positions together with Berberich's positions are

B.M.T. 1900 August 28.5

	α app		δ app	
Extreme Range	$17^h 43^m.0$	$12^h 58^m.5$	-35° 39'	-8° 34'
Berberich	14 47.2	14 02.7	-22 03	-16 58

log a 0.487754

e 0.583751

μ 658".23

P 5.3905 years

If these elements are allowed to undergo the same changes due to a change $d\mu$ in μ as the changes tabulated by Berberich for his elements, the elements corresponding to the limiting periods are

Epoch	1884 August 16.5	1884 August 16.5
P =	5.3353 years	5.4792 years
M =	0° 00' 03".3	359° 59' 09".7
ω =	300 58 42 .7	301 04 51 .1
Ω =	5 13 15 .4	5 05 33 .7
i =	5 26 53 .4	5 28 30 .0
φ =	35 31 46 .2	36 00 22 .3
log a =	0.484773	0.492478
μ =	665".94	647".58

Equinox 1884.0

As a means of comparison with Berberich's ephemeris, positions of the comet were computed from each of these two extreme sets of elements for August 28 and December 2, 1900, the two outer dates of Berberich's search ephemeris. These positions together with Berberich's positions are

G.M.T. 1900 August 28.5

	α_{app}		δ_{app}	
Extreme Range	$17^h 45^m.0$	$12^h 58^m.6$	-35° 39'	-8° 34'
Berberich	14 47.2	14 02.7	-22 03	-16 58

B.M.T. 1900 December 2.5

	α app		δ app	
Extreme Range	$22^h\ 58.0^m$	$17^h\ 07.8^m$	$-4^\circ 56'$	$-26^\circ\ 06'$
Berberich	20 51.1	19 37.1	-19 14'	-24 13

By comparing the extreme ranges in position with those assumed by Berberich, it is seen that the range of the search ephemeris should be considerably larger than that actually used. This increase in extreme range is due both to the allowance made for the errors of observation and also to the fact that the other elements were varied as well as the period. The probable reason why the comet was not found may therefore have been that the comet reappeared in a region outside of the range of the search ephemeris and therefore would not be apt to be found except by accident.

Comet 1881 V (Denning)

Comet 1881 V was discovered by Denning in Bristol on October 4, 1981. It remained under observation until November 24. It was faint at discovery, having reached its maximum brightness in August, and continually decreased in brightness. There were in all 37 observations of October 8,9, and 12. Eight elliptic orbits were computed, the most extensive ones being by Plummer and Matthiessen. The periods range from 7.7 years to 9.1 years. The elements are as follows:[1]

[1] Galle: "Verzeichniss der Elemente der bisher berechneten Cometenbahnen nebst Anmerkungen und Literatur-Nachweisen."

G.M.T. 1900 December 2.5

Extreme Range	22	58.9	17	07.8	-4°56'	-26° 06'
Berberich	20	51.1	19	37.1	-19 14'	-24 13

By comparing the extreme ranges in position with those assumed by Berberich, it is seen that the range of the search ephemeris should be considerably larger than that actually used. This increase in extreme range is due both to the allowance made for the errors of observation and also to the fact that the other elements were varied as well as the period. The probable reason why the comet was not found may therefore have been that the comet reappeared in a region outside of the range of the search ephemeris and therefore would not be apt to be found except by accident.

more observations

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T M.T.1881	ω	☊	i	q	P	Arc	Compute
ptember							
13.67811	313°06'28"	65°41'50"	6°48'11"	0.8323370	9.1	Oct.9-Oct.25	Block
13.08561	312 01 03	66 09 02	6 53 26	0.8240335	8.3	Oct.5-Oct.27	Chandl
13.07707	312 04 18	65 51 34	6 51 45	0.814942	7.7	Oct.5-Oct.18	Schulh
13.25866	312 21 00	65 57 50	6 51 36	0.825348	8.45	Oct.5-Oct.30	Schulh
13.16969	312 11 22	66 04 02	6 52 36	0.824804	8.41	Oct.5-Oct.28	Hartwi
13.4236	312 39 36	65 54 15	6 50 43	0.830188	8.83	Oct.5-Nov.19	and Wutschich Hartwig
13.47214	312 44 11	65 52 02	6 50 23	0.8304135	8.857	Oct.5-Nov.19	Plummer
13.3499	312 30 52	65 56 56	6 51 04	0.828377	8.6874	Oct.9-Nov.21	Matthie

Equinox 1881.0

Plummer's orbit was computed from eight normal places and perturbations were taken into account. His orbit however was not based upon the whole arc of observation. The orbit by Matthiessen[1] was computed from five normal places based upon the whole arc of observation and upon Hartwig's last elements. Matthiessen used Schonfeld's differential formulae[2] for the corrections of the elements on the basis of the residuals in α and δ, using the modification for nearly parabolic orbits. The perturbative effect of the planets Mercury, Venus, the Earth, Mars, Jupiter, and Saturn was considered in the formation of the equations of condition. These perturbations were computed for every four days by the method of the Variation of Constants and the perturbations for the times of the normal places interpolated. The resulting elements are as follows:

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[1] A.N. 121 p. 364

[2] A.N. 2693-95

T 1881 September 13.3499 B.M.T.

$$\left.\begin{array}{ll} \omega & 312^\circ\ 30'\ 52''.1 \\ \Omega & 65\ \ 56\ \ 55\ .6 \\ i & 6\ \ 51\ \ 04.\ 0 \end{array}\right\}\ 1881.0$$

φ 55 55 56 .4

log q 9.860503

log a 0.625927

P 8.6874 years

μ 408".4291

During the time that it was under observation this comet had a very peculiar motion in declination. It remained within a zone of 17 minutes of arc in width in declination, increasing from October 9 to October 14, then decreasing until November 6, and increasing thereafter.

Matthiessen computed a search ephemeris for a return in 1890 using the above elements and taking into account the approximate Jupiter perturbations for the whole revolution between 1881 and 1890. This ephemeris extends from January 17 to August 21, 1890.[1]

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[1] A.N. 123 p. 221 and 124 p. 251.

With this ephemeris corrections are published for a variation of the Perihelion time by one day in either direction. Thus on March 6, the variation in α may be $\mp 2^m.3$, in $\delta \mp 8'.5$. On May 9 these variations are $\mp 3^m.3$ and $\mp 21'.9$. Thus the area for search on March 6 was $1^\circ 9'$ by $17'$, on May 9 $1^\circ 39'$ by $43'.8$. As the comet was

T 1881 September 13.5499 G.M.T.

ω 312° 30' 52".1

Ω 65 56 55 .6   1881.0

i 6 51 04. 0

φ 55 55 56 .4

log q 9.860503

log a 0.625027

P 8.5874 years

μ 408".4291

During the time that it was under observation this comet had a very peculiar motion in declination. It remained within a zone of 17 minutes of arc in width in declination, increasing from October 9 to October 14, then decreasing until November 6, and increasing thereafter.

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quite close to the Sun on this return, there waslittle hope for finding it.

Berberich computed a short search ephemeris for the return in 1898, extending from October 21 to November 26. According to Matthiessen's orbit the perihelion time would be February 10, 1899. This time would make the comet visible during the morning hours. ~~This time would make the comet visible during the morning hours~~. However, Berberich realized the uncertainty of the elements due to a short arc, errors of observation, and indeterminateness in the computation, and therefore varied the perihelion time by 52 days so as to allow for a possibility of finding it during the evening hours. He assumed five perihelion dates 1898 December 20, 1899 January 1, January 13, January 25, and February 6. This makes the area of search 7° by 2° on October 21, and 8° by 0°.7 on November 26. This region of search for November 26 is based upon only the last three perihelion times. There is no record of the comet having been found.

The orbit of this comet will now be investigated by the method used for comet 1884 II. The following normal places of Matthiessen are used:

	B. M.T. 1881	α (1881.0)	δ (1881.0)
I.	October 9.5	143° 29' 39".8	+14° 47' 55".9
II.	October 29.5	153 21 32 .1	14 40 24 .1
III.	November 21.5	160 30 39 .6	+14 53 52 .7

Mathhiessen's orbit leaves the following residuals between observed and computed α and δ in the first and third places:

	I	III
(O - C) $\partial\alpha\cos\delta$	-1".6	+ 0".2
(O - C) $\partial\delta$	-0 .5	-0 .6

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	I	III
(O - C) $\Delta\alpha \cos\delta$	-1".6	+ 0".2
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This is practically a perfect representation. So no attempt will be made to differentially correct Matthiessen's orbit by Leuschner's method to obtain a better representation.

Differential corrections were made by Leuschner's method with the assumption of zero residuals and periods 8.4 and 8.9 years. The residuals resulting from these differential corrections are

Period	8.4 Years I	8.4 Years III	8.9 Years I	8.9 Years III
$(O - C)\, \partial\alpha \cos\delta$	+44".4	~~55".5~~ +37.4	+36".1	+58".3
$(O - C)\, \partial\delta$	-60".7	-37".4	+16".6	-15".1

It is seen that an increase of 0.1 years in the period corresponds to a change of 15".5 in

The expression for $\partial\delta_{\text{I}}$, in terms of $\partial\delta_{\text{III}}$, $\partial\alpha_{\text{I}}$, and $\partial\alpha_{\text{III}}$ in this case is

$$\partial\delta_{\text{I}} = (-1.119)\partial\delta_{\text{III}} + (0.043)\partial\alpha_{\text{I}} + (0.035)\partial\alpha_{\text{III}}.$$

ρ_0 may be regarded constant for the purpose in hand and therefore $K\,\partial\rho_0$ equals zero.

The limiting values for the residuals of the observations used by Matthiessen in the formation of the normal places are as follows:

$$\partial\alpha_{\text{I}} \begin{cases} -0^{s}.19 \\ -1.42 \end{cases} \quad \partial\alpha_{\text{III}} \begin{cases} +0^{s}.43 \\ +0.12 \end{cases} \quad \partial\delta_{\text{I}} \begin{cases} -5''.5 \\ +3\ .6 \end{cases} \quad \partial\delta_{\text{III}} \begin{cases} -2''.2 \\ -1\ .0 \end{cases}$$

The values actually used in the formation of the normal places are

$$\partial\alpha_{\text{I}} = -0^{s}.92 \qquad \partial\alpha_{\text{III}} = +0^{s}.27 \qquad \partial\delta_{\text{I}} = -2.''0 \qquad \partial\delta_{\text{III}} = -1''.6$$

Therefore the departures of the limiting observations from the normal places are

$$\partial\alpha_{\text{I}} \begin{cases} +11''.0 \\ -7\ .5 \end{cases} \quad \partial\alpha_{\text{III}} \begin{cases} +2''.4 \\ -2\ .2 \end{cases} \quad \partial\delta_{\text{I}} \begin{cases} +5.''6 \\ -3.\ 5 \end{cases} \quad \partial\delta_{\text{III}} \begin{cases} +6''.2 \\ -19\ .7 \end{cases}$$

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Period	8.4 Years I	8.4 Years III	8.9 Years I	8.9 Years III
$(O - C)\, \Delta\alpha \cos\delta$	+44".4	~~55".5~~ +37.4	+35".1	+38".3
$(O - C)\, \Delta\delta$	-60 .7	-37 .4	+16 .6	-15 .1

It is seen that an increase of 0.1 years in the period corresponds to a change of 15".5 in

The expression for $\delta\delta_{I}$, in terms of $\delta\delta_{III}$, $\delta\alpha_{I}$, and $\delta\alpha_{III}$ in this case is

$$\delta\delta_{I} = (-1.119)\delta\delta_{III} + (0.048)\delta\alpha_{I} - (0.035)\delta\alpha_{III}.$$

ρ_0 may be regarded constant for the purpose in hand and therefore $K\,\delta\rho_0$ equals zero.

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$$\delta\alpha_{I}\left\{\begin{matrix}-0^{s}.19\\-1.42\end{matrix}\right. \qquad \delta\alpha_{III}\left\{\begin{matrix}+0^{s}.43\\+0.12\end{matrix}\right. \qquad \delta\delta_{I}\left\{\begin{matrix}-5''.5\\+3.6\end{matrix}\right. \qquad \delta\delta_{III}\left\{\begin{matrix}-2''.8\\-1.0\end{matrix}\right.$$

The values actually used in the formation of the normal places are

$$\delta\alpha_{I} = -0^{s}.92 \qquad \delta\alpha_{III} = +0^{s}.27 \qquad \delta\delta_{I} = -2''.0 \qquad \delta\delta_{III} = -1''.8$$

Therefore the departures of the limiting observations from the normal places are

+11".0 +2".4 +5".6 +6".2

Therefore the greatest negative change in $\partial\delta_1$, due to a possibility of the limiting observations being the correct ones would be represented by

$(-1.119)(6''.2)+(0.043)(-7''.5)+(-0.035)(2''.4) -3''.5 = -10''.8$

The greatest positive change would be

$(-1.119)(-19''.7)+(0.043)(11''.0)+(-0.035)(-2''.2)+ 5''.6= +28''.2$

Since there is a range of over a second of time in the right ascension of the middle place, the inaccuracy of which has been neglected in this case as in the case of the orbit of Comet 1884 II, these changes in $\partial\delta_1$, may be called roughly -15" and +30". Since a change of + 15".5 in $\partial\delta_1$, corresponds to a change of +0.1 years in the period, the change of +30" corresponds to a change of +0.194 years and the -15" to a change of -0.097 years in the period. This makes an extreme range of 0.291 years or 106.2 days as compared with the range of 2 days allowed in the perihelion time for the 1890 emphemeris and of 312 days as compared with the range of 52 days allowed in the 1899 ephemeris.

In view of the large residuals in the right ascensions of the first and third places and in the declination of the third place resulting from the attempt to force periods of 8.4 and 8.9 years, it was deemed advisable to represent these places by means of Matthiessen's orbit. The representation yielded the surprising residuals

	I	III
$(O - C)\ \partial\alpha\cos\delta$	+ 36.#1	+ 50."6
$(O - C)\ \partial\delta$	-16. 3	-24. 1

Therefore the greatest negative change in δ, due to a possibility of the limiting observations being the correct ones would be represented by

$$(-1.119)(6''.2) + (0.043)(-7''.5) + (-0.035)(2''.4) - 3''.5 = -10''.8$$

The greatest positive change would be

$$(-1.119)(-19''.7) + (0.043)(11''.0) + (-0.035)(-2''.2) + 5''.6 = +23''.2$$

Since there is a range of over a second of time in the right ascension of the middle place, the inaccuracy of which has been neglected in this case as in the case of the orbit of Comet 1884 II, these changes in δ, may be called roughly -15" and + 30". Since a change of + 15".5 in δ, corresponds to a change of + 0.1 years in the period, the change of + 30" corresponds to a change of + 0.194 years and the -15" to a change of -0.097 years in the period. This makes an extreme range of 0.291 years or 106.2 days as compared with the range of 2 days allowed in the perihelion time for the 1890 ephemeris and of 312 days as compared with the range of 52 days allowed in the 1899 ephemeris.

In view of the large residuals in the right ascensions of the first and third places and in the declination of the third place resulting from the attempt to force periods of 8.4 and 8.9 years, it was deemed advisable to represent these places by means of Matthiessen's orbit. Therepresentation yielded the surprising residuals

	I	III
(O - C) $\Delta\alpha \cos\delta$	+ 36".1	+ 50".6
(O - C) $\Delta\delta$	-16. 3	-24. 1

A differential correction to remove these residuals resulted in a period of 8.809 years. A comparison of this period with those of the orbits tabulated shows it to be in close agreement with the periods of the orbit by Plummer and that by Hartwig. This, together with the close agreement between those two orbits, makes it advisable to assume as the most probable period the mean of the periods 8.83 and 8.857 years. This mean is 8.844 years. Applying the variations of -0.097 years and +0.194 years to this period gives 8.747 and 9.038 years as the approximate limiting values of the period. This range is seen to include three of the orbits tabulated, while the orbits by Block and Matthiessen are just beyond the limits.

The elements corresponding to these limiting periods are

Epoch	1881 October 29.5 B.M.T.	October 29.5
P	8.747 years	9.038 years
M	5° 11' 06"	4° 53' 57"
ω	313 36 13	314 31 33
☊	65 05 41	64 35 58
i	6 44 48	6 41 07
φ	55 49 18	56 16 36
log a	0.62791	0.63738
μ	405".65	392.59

Equinox 1881.0

The search ephemeris for 1890 gives the following position for 1890 January 17.5 B.M.T.

$$\alpha_{app}\ 18^h\ 36^m\ 12^s \mp 1^m.3 \qquad \delta_{app}\ -24\ 59'.6 \pm 9'.9$$

the variations $\mp$ $1^m.3$ and $\pm 9'.9$ corresponding to a change of the

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Epoch	1881 October 29.5 E.M.T.	October 29.5
P	8.747 years	9.038 years
M	5° 11' 08"	4° 53' 57"
ω	313 36 13	314 31 33
☊	65 06 41	64 35 58
i	6 44 48	6 41 07
φ	55 49 18	56 16 36
log a	0.62791	0.63738
μ	405".65	392.59

Equinox 1881.0

The search ephemeris for 1890 gives the following position for 1890 January 17.5 E.M.T.

α_{app} $15^h 56^m 12^s \mp 1^m.3$ δ_{app} $-24\ 59'.5 \pm 9'.9$

the variations $\mp 1^m.3$ and $\pm 9'.9$ corresponding to a change of the

perihelion time by 1 day. For the sake of comparison, positions for 1890 January 17.5 were computed with the two limiting sets of elements. These positions are

	$P = 8.747$ years	$P = 9.038$ years
α app	$18^h\ 36^m\ 58^s$	$16^h\ 54^m\ 07^s$
δ app	$-24^\circ\ 59'.0$	$-22^\circ\ 25'.9$

Thus on January 17.5 the comet could lie anywhere in an area $25^\circ 43'$ by $2^\circ 33'$ as compared with the area $39'.0$ by $19'.8$ allowed by Matthiessen. The failure to find the comet may have been due to the fact that it lay outside the area allowed for in the search ephemeris. It thus seems that more allowance should have been made for errors in observations and for indeterminateness, and that the other elements should have been varied as well as the period.

In A.N. 101 p.78 Winnecke mentions the possibility of the identity of this comet with one discovered by Goldschmidtt on May 16, 1855, in the position

$$\alpha = 21^h\ 14^m\ 46^s, \qquad \delta = -15\ \ 38'.$$

Winnecke found that if Denning's comet passed through perihelion on August 5, 1855, then according to the orbit by Hartwig and Wutschichowski on May 16, 1855, it would be in the position $\alpha = 325^\circ$, $\delta = -19^\circ$. This difference in position would change the period to 8.72 years. He accounted for this difference by the fact that if these comets are identical the comet would have passed through perihelion on January 12, 1873 within 0.02 astronomical units of Venus. The period 8.72 years lies close to the lower limit~~s~~ found above. This fact seems to add to the suspicion as to the identity of the two comets.

perihelion time by 1 day. For the sake of comparison, positions for 1900 January 17.5 were computed with the two limiting sets of elements. These positions are

	P = 8.747 years	P = 9.038 years
α app	$13^h\ 26^m\ 53^s$	$15^h\ 54^m\ 07^s$
δ app	$-24^\circ\ 59'.0$	$-22^\circ\ 25'.9$

Thus on January 17.5 the comet could lie anywhere in an area $25^\circ 43'$ by $2^\circ 33'$ as compared with the area $39'.0$ by $19'.8$ allowed by Matthiessen. The failure to find the comet may have been due to the fact that it lay outside the area allowed for in the search ephemeris. It thus seems that more allowance should have been made for errors in observations and for indeterminateness, and that the other elements should have been varied as well as the period.

In A.N. 101 p.78 Winnecke mentions the possibility of the identity of this comet with one discovered by Goldschmidt on May 16, 1855, in the position

$$\alpha = 21^h\ 14^m\ 46^s, \qquad \delta = -15\ 39'.$$

Winnecke found that if Denning's comet passed through perihelion on August 5, 1855, then according to the orbit by Hartwig and Wotschichowski on May 16, 1855, it would be in the position $\alpha = 325^\circ$, $\delta = -19^\circ$. This difference in position would change the period to 8.72 years. He accounted for this difference by the fact that if these comets are identical the comet would have passed through perihelion on January 12, 1873 within 0.02 astronomical units of Venus. The period 8.72 years lies close to the lower limit found above. This fact seems to add to the suspicion as to the identity of the two comets.

From the investigations on these two comets it can be concluded that the failure of certain comets to be found on their predicted returns is in many cases probably due to an altogether too limited range of the search ephemerides, the elements being considerably less accurate than the computers have supposed on the basis of their unnecessarily refined investigations.

The writer wishes to express her sincere appreciation to Director A.O.Leuschner, Professor R.T.Crawford, and others of the Students' Observatory of the University of California for their kind advice during the progress of this work, and to Mr.William C.James of Evanston, Illinois, and Mrs. Edith Quimby of New York, for duplicating a part of the computation.

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